The Leslie-Gower competition model with fast dispersal

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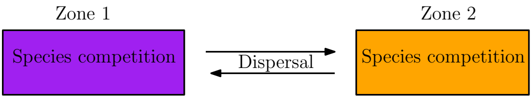
- **Dispersal-competition tradeoff?**
- **Does fast dispersal yield new dynamic scenarios?**
  The Leslie-Gower competition model (discrete) allows
  - Species coexistence.
  - One species extinction (unconditionally).
  - One species extinction (depending on initial values).

Model’s settings



Notation

S: competition, demography

F: dispersal

$$S(x,y)=\begin{pmatrix}\dfrac{b_1}{1+c_{11}x+c_{12}y}x\\ \dfrac{b_1}{1+c_{21}x+c_{22}y}y\end{pmatrix}$$

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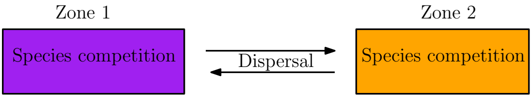

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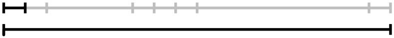

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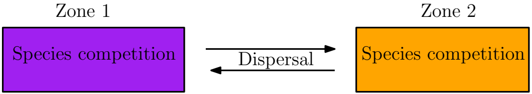

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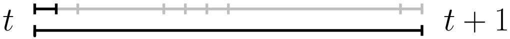
We choose the slow time unit t



$$N(t+1) = S \circ \overbrace{F \circ F \circ \cdots F}^{k \text{ times}} (N(t)) = S \circ F^{(k)} (N(t))$$

The equations

Notation: $N = (\overbrace{N_1^1, N_1^2}^{sp1\ N_1}, \overbrace{N_2^1, N_2^2}^{sp2\ N_2}), \qquad n_i = N_i^1 + N_i^2$

Fast dynamics: constant dispersal rates $0 < p_i, q_i < 1$

$$F(N) = \begin{pmatrix} \mathcal{F}_1 & 0 \\ 0 & \mathcal{F}_2 \end{pmatrix} N, \qquad \text{where} \qquad \mathcal{F}_i = \begin{pmatrix} 1 - p_i & q_i \\ p_i & 1 - q_i \end{pmatrix}$$

Slow dynamics: local Leslie-Gower competition model $b_i^j, c_{ij}^k > 0$

$$S(N) = \left(\frac{b_1^1 N_1^1}{1+c_{11}^1 N_1^1 + c_{12}^1 N_2^1}, \frac{b_1^2 N_1^2}{1+c_{11}^2 N_1^2 + c_{12}^2 N_2^2}, \frac{b_2^1 N_2^1}{1+c_{21}^1 N_1^1 + c_{22}^1 N_2^1}, \frac{b_2^2 N_2^2}{1+c_{21}^2 N_1^2 + c_{22}^2 N_2^2} \right)^T$$

$$N(t+1) = S \circ F^{(k)} \left(N(t)\right)$$

Approximate aggregation $N(t+1) = S \circ F^{(k)}(N(t))$

Given that $\lim_{k\to\infty} \mathcal{F}_i^k N_i = \begin{pmatrix} \nu_i^* \\ 1-\nu_i^* \end{pmatrix} n_i$ where $\nu_i^* = \dfrac{q_i}{p_i+q_i}$
yields the so-called *auxiliary system*

$$N(t+1) = S \circ \bar{F}(N(t))$$

A center manifold result yields the *aggregated system*:

$$\begin{cases} n_1(t+1) = f(n_1(t), n_2(t))n_1(t) \\ n_2(t+1) = g(n_1(t), n_2(t))n_2(t) \end{cases}$$

where

$$f(n_1,n_2) = \left(\frac{b_1^1\nu_1^*}{1+c_{11}^1\nu_1^* n_1 + c_{12}^1\nu_2^* n_2} + \frac{b_1^2(1-\nu_1^*)}{1+c_{11}^2(1-\nu_1^*)n_1 + c_{12}^2(1-\nu_2^*)n_2} \right)$$

$$g(n_1,n_2) = \left(\frac{b_2^1\nu_2^*}{1+c_{21}^1\nu_1^* n_1 + c_{22}^1\nu_2^* n_2} + \frac{b_2^2(1-\nu_2^*)}{1+c_{21}^2(1-\nu_1^*)n_1 + c_{22}^2(1-\nu_2^*)n_2} \right)$$

Approximate aggregation $N(t+1) = S \circ F^{(k)}(N(t))$

General results

Let $n^* = (n_1^*, n_2^*) \in \mathbb{R}^2$ be a hyperbolic fixed point of the reduced system. Then

1. For all $k \geq k_0$ the slow-fast system has an unique fixed point N_k^* which is hyperbolic and

$$\lim_{k\to\infty} N_k^* = (\nu_1^* n_1^*,\ (1-\nu_1^*)n_1^*,\ \nu_2^* n_2^*,\ (1-\nu_2^*)n_2^*)$$

2. n^* is A.S. (U.) $\Leftrightarrow$ N_k^* is A.S. (U.) for all $k \geq k_1$.

3. The basin of attraction of N_k^* can be described from that of n^*.

* L. Sanz, R. Bravo de la Parra, E. Sánchez (2008). *Two time scales non-linear discrete models approximate reduction*. Journal of Difference Equations and Applications, 14, No. 6, 607–627.

Results from the aggregated system:

All solutions in $[0, \infty) \times [0, \infty)$ converge to an equilibrium in an eventually monotonic manner

- Explicit expressions for fixed points $(n_1^*, 0), \ (0, n_2^*)$.
- Explicit conditions for the stability of $(n_1^*, 0), \ (0, n_2^*)$.
- Complicated expression for coexistence equilibrium.
- Spatial homogeneity: outcomes as in the Leslie-Gower model, but **dispersal does matter**!!
- Spatial heterogeneity: **up to 3 coexistence equilibrium points**!

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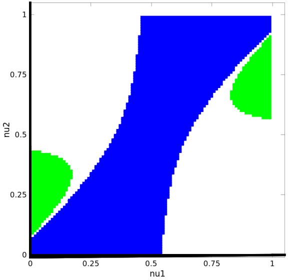
Dispersal-competition tradeoff?

- Spatial homogeneity $c_{12}^1 = c_{12}^2$, $c_{21}^1 = c_{21}^2$, $c_{ii}^1 = c_{ii}^2$, $b_1^1 = b_1^2 = b_2^1 = b_2^2$.
- In absence of dispersal: species 1 outcompetes species 2.

$$b = 1.5, \mathbf{c_{21} = 1.1}, \mathbf{c_{12} = 0.9}$$



Blue	species 2 extinction
White	global coexistence
Green	species 1 extinction

Dispersal-competition tradeoff? spatial homogeneity

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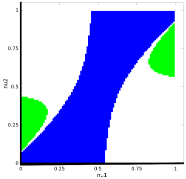
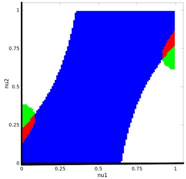
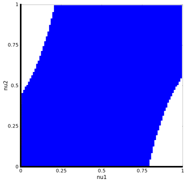


$b = 1.5$, $\mathbf{c_{21} = 1.5}$, $\mathbf{c_{12} = 0.85}$



$b = 1.5$, $\mathbf{c_{21} = 3}$, $\mathbf{c_{12} = 0.7}$

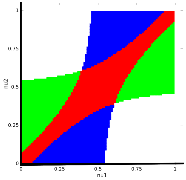


Red	conditional extinction
White	global coexistence
Green	species 1 extinction
Blue	species 2 extinction

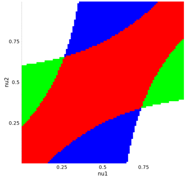
Dispersal-competition tradeoff? spatial homogeneity

- In absence of dispersal: conditional extinction.

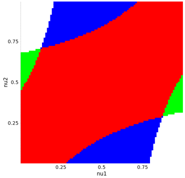
$b = 1.5, \mathbf{c_{21} = 1.1}, \mathbf{c_{12} = 1.1}$



$b = 1.5, \mathbf{c_{21} = 1.5}, \mathbf{c_{12} = 1.3}$



$b = 1.5, \mathbf{c_{21} = 3}, \mathbf{c_{12} = 1.7}$



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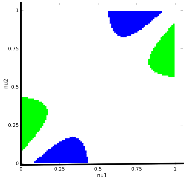
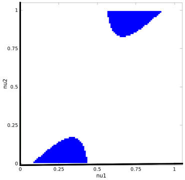
Dispersal-competition tradeoff? spatial homogeneity

- In absence of dispersal: coexistence.

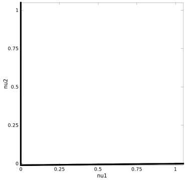
$b = 1.5$, $\mathbf{c_{21} = 0.9}$, $\mathbf{c_{12} = 0.9}$



$b = 1.5$, $\mathbf{c_{21} = 0.9}$, $\mathbf{c_{12} = 0.7}$



$b = 1.5$, $\mathbf{c_{21} = 0.7}$, $\mathbf{c_{12} = 0.7}$



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Green	species 1 extinction
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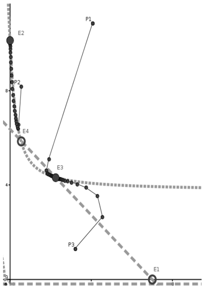
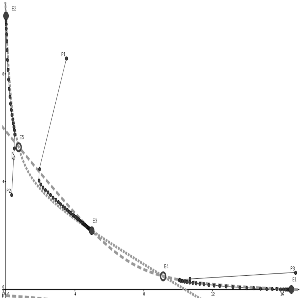
Spatial heterogeneity allows new dynamics

* At most three coexistence equilibria.





Conclusions

1. There exists a trade-off between competition and (fast) dispersal.

2. Fast dispersal between heterogeneous regions may lead to new dynamical scenarios.

Thank you; questions?