On the Beddington-DeAngelis competitive response

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Abstract The Competitive Exclusion Principle [5] establishes that two species competing for the same resource can not coexist. Gause's interference competition model [5] supports this idea: when competing, one species will not be driven to extinction if the inter-species competition pressure is lower than the intra-species competition pressure. The so-called **competitive strength** conveys this idea

$$\begin{cases} x_1' = r_1 x_1 - a_{11}x_1^2 - a_{12}x_1x_2 \\ x_2' = r_2 x_2 - a_{22}x_2^2 - a_{21}x_1x_2 \end{cases} \Leftrightarrow \quad u_i = \frac{a_{ii}u_i}{r_i}, \quad c_{ij} = \frac{a_{ij}/r_i}{a_{jj}/r_j}$$

competitive strength of species j on species i

$$\Leftrightarrow (1) \begin{cases} u_1' = r_1 u_1(1 - u_1 - c_{12}u_2) \\ u_2' = r_2 u2(1 - u_2 - c_{21}u_1) \end{cases}$$

where:

- r_i growth rate
- a_{ii} intra-species competition
- a_{ij} competition effect of species j on i

species i cannot exclude species j iif $c_{ij} < 1$

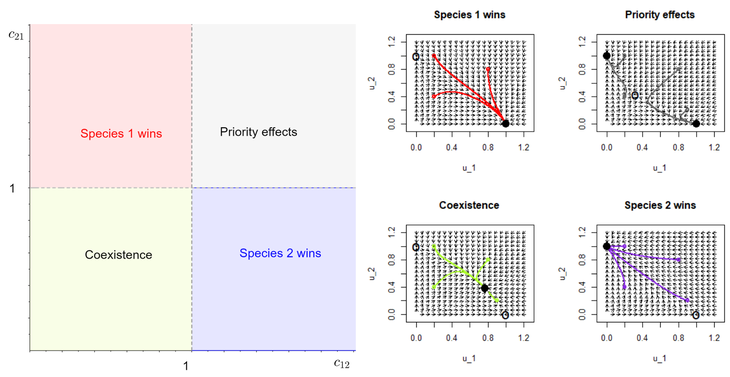
The Coexistence Paradox points out that the Competitive Exclusion Principle is at odds with reality: species coexist much more than predicted by Gause's Principle. We **incorporate to the classical competition model (1) the time invested in competitor's mutual interference**, and compare our results to previous research [3], [4].

Classical model [5]

Time allowance

Searching, feeding, mating, ...

$$(1) \begin{cases} u_1' = r_1 u_1(1 - u_1 - c_{12}u_2) \\ u_2' = r_2 u2(1 - u_2 - c_{21}u_1) \end{cases}$$

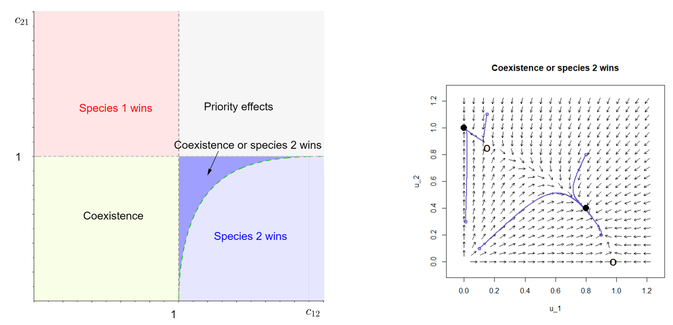


Holling II [3]

Time allowance

Searching, feeding, mating, ...	Inter-species competition

$$(2) \begin{cases} u_1' = r_1 u_1 \left(1 - x_1 - \dfrac{c_{12}u_2}{1 + c_1 u_1}\right) \\ u_2' = r_2 u2\,(1 - u_2 - u_{21}u_1) \end{cases}$$

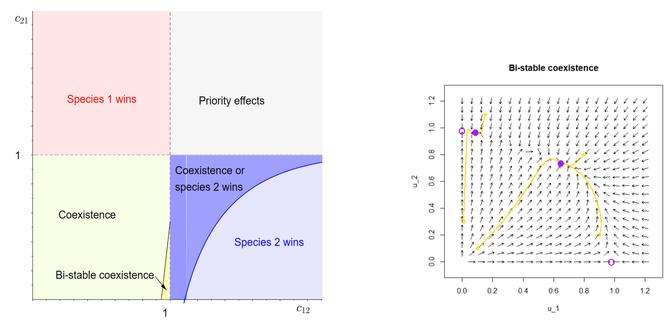


Holling IV [4]

Time allowance

Searching, feeding, mating, ...	Inter-species competition increases linearly with u_1

$$(3) \begin{cases} u_1' = r_1 u_1 \left(1 - u_1 - \dfrac{c_{12}u_2}{1 + c_1 u_1^2}\right) \\ u_2' = r_2 u2(1 - u_2 - c_{21}u_1) \end{cases}$$



The Beddington-DeAngelis competitive response

The ideas stated [1, 2] for predator-prey models apply for species competition. We account for

- The time spent in competition by individuls of species 2 when dealing with individuals of species 1: c_1
- The time spent in mutual interference by individuals of species 2 when competing individuals of species 1: $\tilde{a}_2 \in (0, 1)$ and $\tilde{c}_2$.

Let us consider that only species 2 exhibits the Beddington-DeAngelis competitive response: interactions are time-consumming

Time allowance

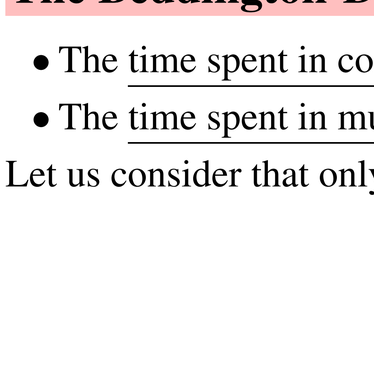
Searching, seeding, mating, ...	Inter-species competition	Intra-species 2 interference

$$\begin{cases} u_1' = r_1 u_1 \left(1 - u_1 - \dfrac{c_{12}u_2}{1 + c_1 u_1 + \tilde{a}_2(\tilde{c}_2 u_2 - 1)}\right) \\ u_2' = r_2 u_2\,(1 - u_2 - c_{21}u_1) \end{cases}$$

Assuming $\tilde{c}_2 > 1$:

1. Any $(c_{12}, c_{21}) \in [1,\ 1 + \tilde{a}(\tilde{c}_2 - 1)] \times [0,\ 1]$ leads to coexistence, improving coexist to the classical (1), Holling type II (2) and type IV (3) competition models.
2. There is a wider range of (c_{12}, c_{21}) values leading species 1 to unconditionally remove species 2.

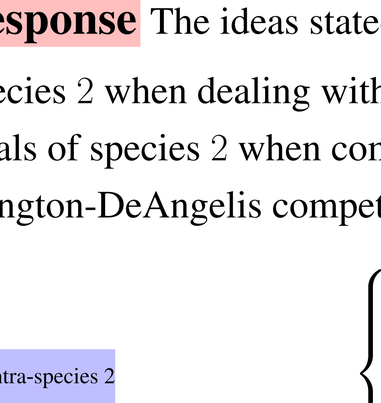
Competing time smaller than interference time



Dark red region:

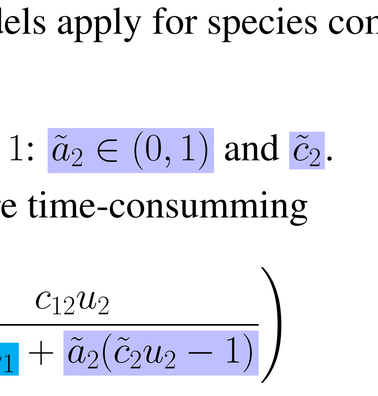
- Is similar to that in (2) and (3).
- Species coexist or species 1 wins.
- Species 2 may survive where otherwise would die.

Competing time equal to interference time



- No conditional coexistence is allowed.

Competing time larger than interference time



Dark purple region:

- Is as in (2) or (3).
- Species coexists or species 2 wins.
- Species 1 may survive where otherwise would die.

References

[1] Beddington, J. R. (1975). Mutual interference between parasites or predators and its effect on searching efficiency. *J. Anim. Ecol.*, 331-340.

[2] DeAngelis, D. L., Goldstein, R. A., O'Neill, R. V. (1975). A model for tropic interaction. *Ecol.*, 56(4), 881-892.

[3] Castillo-Alvino, H., Marvá, M. (2020). The competition model with Holling type II competitive response to interfering time. *J. Biol. Dyn.*, 14(1), 222-244.

[4] Castillo-Alvino, H., Marvá, M. (2022). Group defense promotes coexistence in interference competition: The Holling type IV competitive response. *Math. Comp. Simul.*, 198, 426-445.

[5] Gause, G. (1935), The struggle for existence, Annals of the Entomological Society of America. 28(1) 59

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