Competencia entre especios, interferencia entre individuos y coexistencia.

¿Qué dicen los modelos deterministas?

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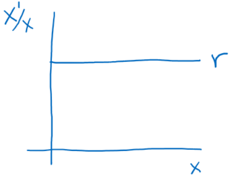
Towards the classical competition model

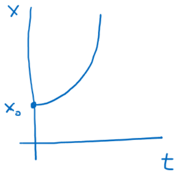
- $x(t)$ the total population size at time t.
- $x'(t)$ the variation of the total population size at time t.
- $\dfrac{x'(t)}{x(t)}$ the *per capita* growth rate.

Malthus's model [1]

The per capita growth rate is constant

$$\frac{x'(t)}{x(t)} = r \quad \Leftrightarrow \quad x'(t) = rx(t) \quad \Leftrightarrow \quad x(t) = x_0 e^{rt}, \quad x_0 = x(0), \text{ initial population size}$$





Towards the classical competition model

Verlhust's model [2]

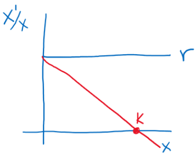
The growth rate decreases linearly as the population grows until reaching the carrying capacity

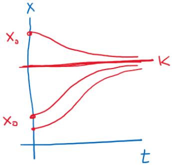
$$\frac{x'(t)}{x(t)} = r\left(1 - \frac{x}{K}\right)$$

$$\Leftrightarrow\ x'(t) = rx(t) - r\frac{x^2(t)}{K}$$

$$\Leftrightarrow \quad x(t) = \tfrac{K}{1+(K-x_0)e^{-rt}}, \quad x_0 = x(0)$$

- r is the intrinsic growth rate.
- K is the carrying capacity.
- The $x^* = 0$ and $x^* = K$ are **equlibrium states**.
- r/K is the intra-species competition coefficient.





Note that

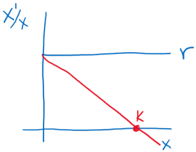
$$\frac{x'(t)}{x(t)} = r\left(1 - \frac{x}{K}\right)$$

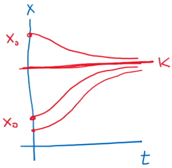
can be written as

$$\frac{x'(t)}{x(t)} = r - a_{11}x$$

where $a_{11} = \dfrac{r}{K}$ stands for the intra-species competition coefficient, so that

- The per capita growth rate decreases as the population growth due to intra-species competition.





Towards the classical competition model

- $x_1(t)$ the total population size of species 1 at time t.
- $x_2(t)$ the total population size of species 2 at time t.

The classical INTERFERENCE competition model (Gausse [3]

The per capita growth rate linearly decreases as:

- the population size grows until reaching the carrying capacity.
- the competitors population size grows

$$\begin{cases} \frac{x_1'(t)}{x_1(t)} = r_1\left(1 - \frac{x_1}{K_1}\right) - b_{12}x_2 \\ \\ \frac{x_2'(t)}{x_2(t)} = r_2\left(1 - \frac{x_2}{K_2}\right) - b_{21}x_1 \end{cases}$$

Besides

- Logistic growth in the absence of competitors.
- b_{ij} competition coefficient.
- No formula for solutions. Qualitative study.

The classical competition model

Rearranging terms

$$\begin{cases} x_1' = \overbrace{r_1 x_1 - a_{11} x_1^2}^{\text{intra species dynamics}} - \overbrace{a_{12} x_1 x_2}^{\text{inter species interactions}} \\ x_2' = r_2 x_2 - a_{22} x_2^2 - a_{21} x_1 x_2 \end{cases}$$

where

- r_i is the intrinsic growth rate.
- a_{ii} is the intra-species competition.
- a_{ij} effect of species j on species i: inter-species competition.

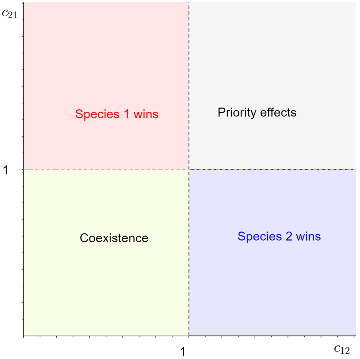
There are 4 equilibrium states:

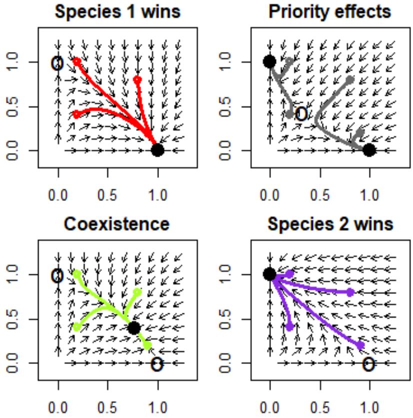
1. $E_0^* = (0,0)$ both populations extinction.
2. $E_1^* = (x_1^*, 0)$ species 2 extinction.
3. $E_2^* = (0, x_2^*)$ species 1 extinction.
4. $E_3^* = (x_1^*, x_2^*)$ species coexistence.

Define

$$c_{12} = \frac{a_{12}}{a_{22}} \frac{r_1}{r_2} \qquad c_{21} = \frac{a_{21}}{a_{11}} \frac{r_2}{r_1}$$

- $c_{12} < 1 \Leftrightarrow$ species 1 can not go extinct.
- $c_{21} < 1 \Leftrightarrow$ species 2 can not go extinct.





Limitations of the classical competition model:

1. Assumes that inter species interactions do not take time.

   Time allowance

   Searching, feeding, mating,...

2. Assumes that individuals are well mixed.

3. Coexistence's Paradox: Small region in the parameters space enabling coexistence.

We next...

1. ...introduce further extensions of the classical competition model.

2. For the sake of simplicity, only species 1 displays the new effects.

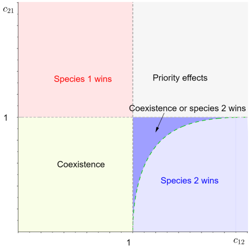
Time allowance

Searching, feeding, mating,...	Inter-species competition

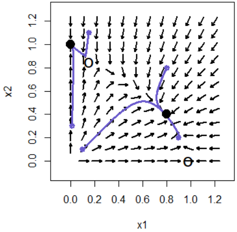
$$\begin{cases} x_1' = r_1 x_1 - a_{11}x_1^2 - \dfrac{a_{12}x_1x_2}{1 + a_1 x_1} \\ x_2' = r_2 x_2 - a_{22}x_2^2 - a_{21}x_1x_2 \end{cases} \quad (1)$$

- a_1: x_2 spends time in competing with x_1.
- a_1 = resources finding rate × probability of finding a competitor × time spent competing.

$a_1 > 0$ in (1)



Conditional coexistence in favor of sp 1



Holling type IV competitive response: group defense [5]

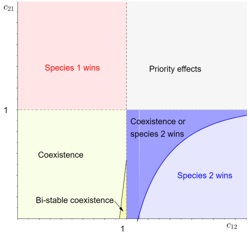
Time allowance

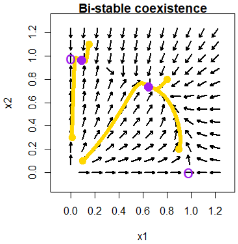
Searching, feeding, mating,...	Inter-species competition

$$\begin{cases} x_1' = r_1 x_1 - a_{11} x_1^2 - \dfrac{a_{12} x_1 x_2}{1 + a_1 x_1^2} \\ x_2' = r_2 x_2 - a_{22} x_2^2 - a_{21} x_1 x_2 \end{cases} \qquad (2)$$

- c_1: x_2 spends time when dealing with x_1.
- The more x_1, the more time spends x_2 in competition.
- a_1 = resources finding rate × probability of finding a competitor × time spent competing.

$a_1 > 0$ in (2)





Time allowance

Searching, feeding, mating, ...	Inter-species competition	Intra-species 2 interference

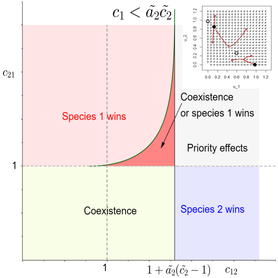
- c_1 : x_2 spends time when competing with x_1.
- $\tilde{a}_2$: x_2 spends time interfering with x_2 when competing with x_1.
- $\tilde{a}_2$: species 2 conspecifics' encounter rate $\times$ intra-species 2 average interference time.

$$\begin{cases} x_1' = x_1 x_1 - a_{11} x_1^2 - \dfrac{a_{12} x_1 x_2}{1 + c_1 x_1 + \tilde{a}_2 (x_2 - 1)} \\ x_2' = r_2 x_2 - a_{22} x_2^2 - a_{21} x_1 x_2 \end{cases} \tag{3}$$

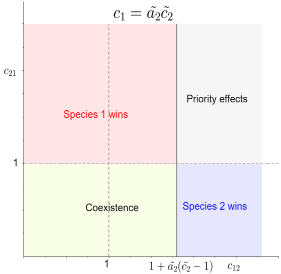
Assume $c_1 > 0$ and $\tilde{a}_2 > 0$ in (3).

$$c_{12} = \frac{a_{12}}{a_{22}} \frac{r_1}{r_2} \qquad c_{21} = \frac{a_{21}}{a_{11}} \frac{r_2}{r_1} \qquad \tilde{c}_i = \frac{r_i}{a_{ii}} \text{ carrying capacity}$$

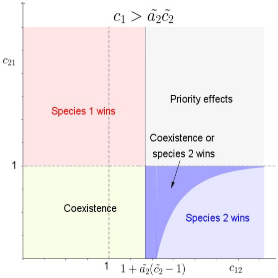
Competing time smaller than interference time



Competing time equal to interference time



Competing time larger than interference time



Holling type IV competitive response: group defense [5]

Conclusions

Taking into account inter- and intra-species interference in the competition term

- Yields new coexistence scenarios.
- Enlarges the coexistence region in the parameters space.

Next steps

:

- Complete the analysis of the Beddington-DeAngelis competition model.
- Compare the above models to real data.
- Translate the above ideas to discrete (difference equation) models.

THANK YOU

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